



#### **Magic of Models**

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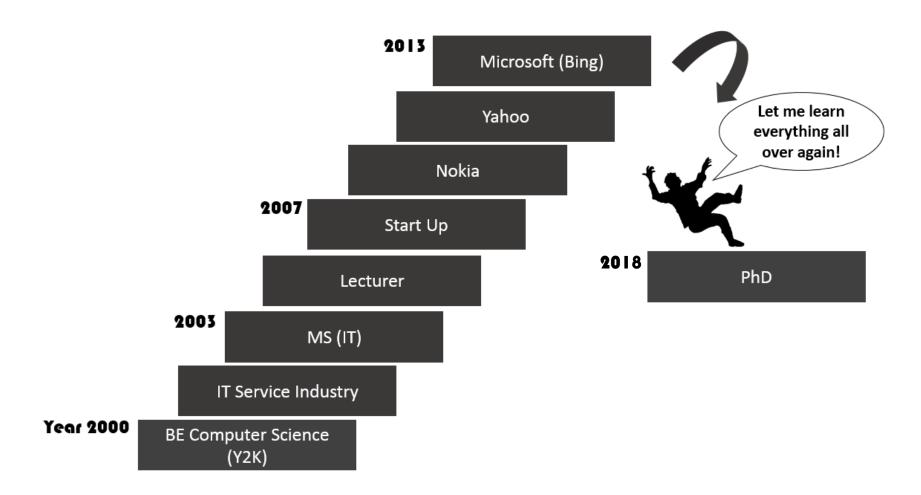
Venkatesh Vinayakarao



3:00 PM - 4:00 PM Agilisium Consulting Oval Conference Room

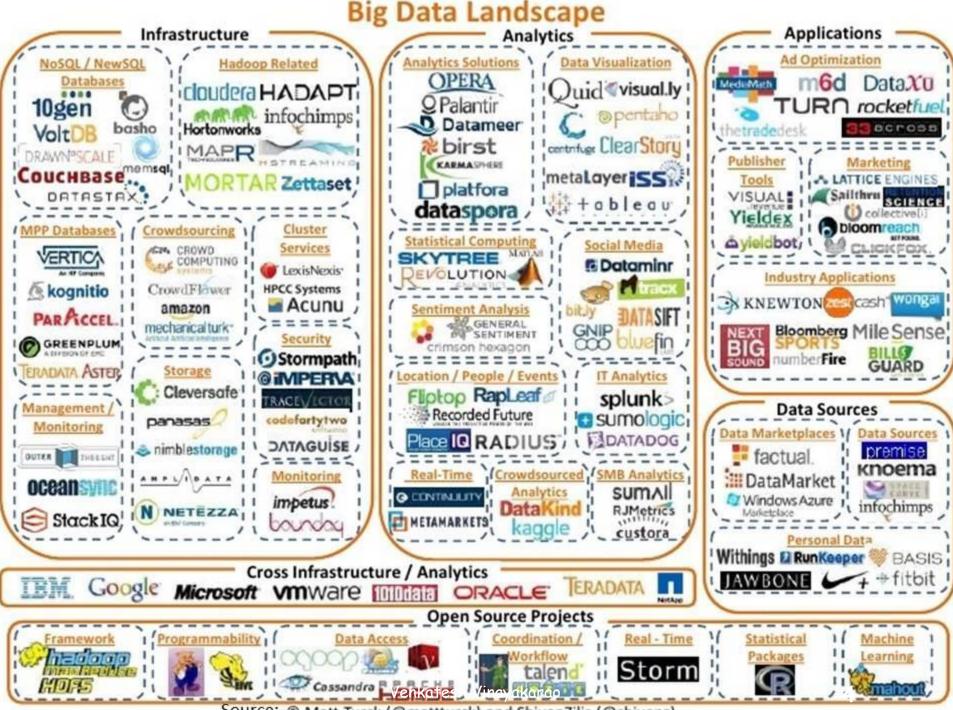
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#### About Me



# Big Data is Ubiquitous





Source: © Matt Turck (@mattturck) and ShivonZilis (@shivonz)

To make sense out of the data...

#### How to tame complexity?

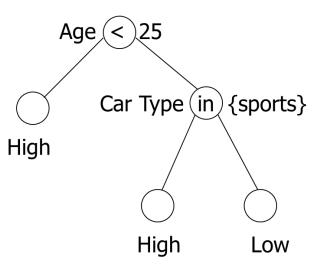
Answer: Design effective models!

## A Simple Model

Data Set		
Age	Car Type	Risk
23	Family	High
17	Sports	High
43	Sports	High
68	Family	Low
32	Truck	Low
20	Family	High

Question: What is the risk (high or low) if age is below 25?

#### **Decision Tree**



# Agenda

- Case Study 1: How to make decisions based on data?
  - Bayesian Data Analysis
- Case Study 2: How to effectively retrieve relevant documents?
  - Vector Space Models



Thomas Bayes, 1701 to 1761

#### Bayesian Data Analysis and Beta Distribution

Venkatesh Vinayakarao

# The Case of Coin Flips

General Assumption: If a coin is fair! Heads (H) and Tails (T) are equally likely. But, coin need not be fair  $\circledast$ 

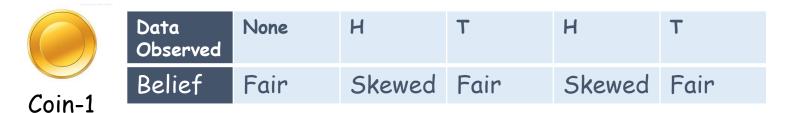


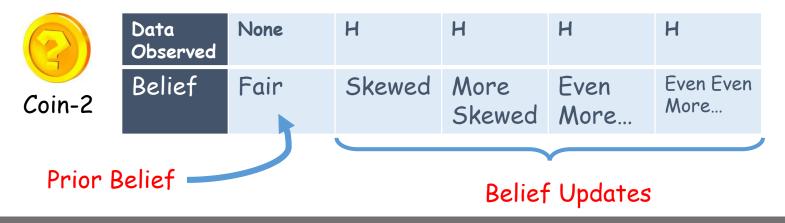
Experiment with Coin - 1 HHHTTTHTHT Experiment with Coin - 2 HHHHHHHHT

Coin-1 more likely to be fair when compared to coin-2.

## Our Beliefs

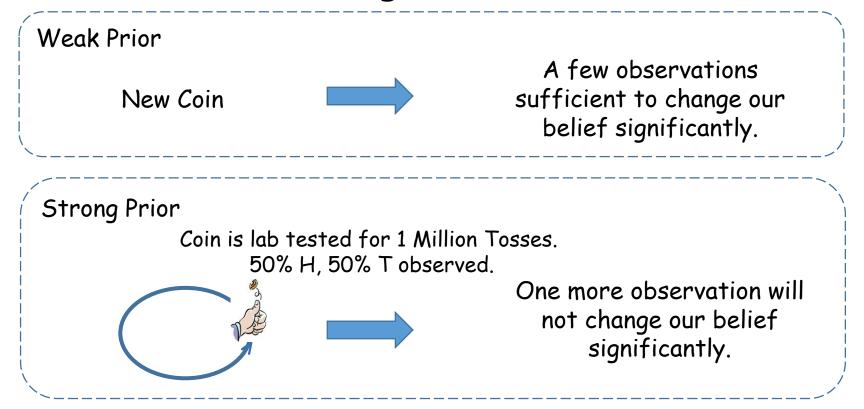
• Can we find a structured way to determine coin's nature?





### Priors

Priors can be strong or weak



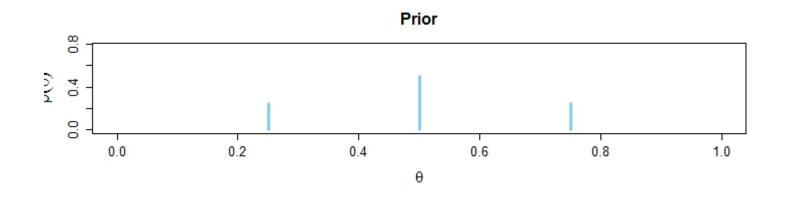
# HyperParameter

- Prior probability (of Heads) could be anything:
  - O.5 → Fair Coin
  - 0.25  $\rightarrow$  Skewed towards Tails
  - 0.75  $\rightarrow$  Skewed towards Heads
  - 1  $\rightarrow$  Head is guaranteed!
  - $0 \rightarrow$  Both sides are Tails.

We use  $\theta$  as a HyperParameter to visualize what happens for different values.

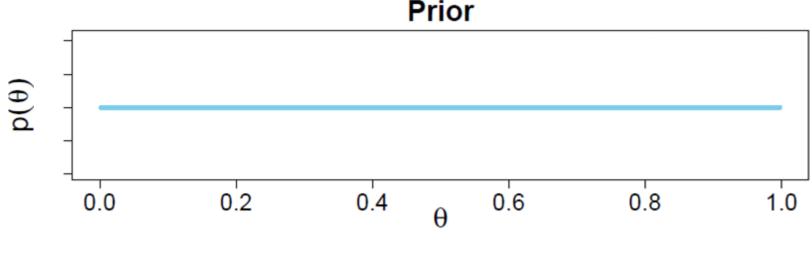
## World of Distributions

#### Discrete Distribution of Prior. Since I typically perceive coins as fair, Prior belief peaks at 0.5.



## Another Possibility

I may also choose to be unbiased! i.e.,  $\theta$  may take any value equally likely.



A Continuous Uniform Distribution!

#### Observations

Let's flip the coin (N) 5 times. We observe (z) 3 Heads.

## Impact of Data

# Belief is influenced by Observations. But, note that:

#### Belief ≠ observation



Bayes' Rule  

$$Pr(\theta \mid D) = \frac{Pr(D \mid \theta) Pr(\theta)}{Pr(D)}$$

#### Numerator is easy

- $p(\theta)$  was uniform. So, nothing to calculate.
- How to calculate  $p(D|\theta)$ ?



Jacob Bernoulli 1655 - 1705.

$$\theta^{z}(1-\theta)^{n-z}$$

If D observed is HHHTT and  $\theta$  is 0.5, We have:  $p(D|\theta) = (0.5)^3(1-0.5)^{5-3}$ 

Remember, two things: 1)we are interested in the distribution 2) Order of H,T does not matter.

#### Painful Denominator

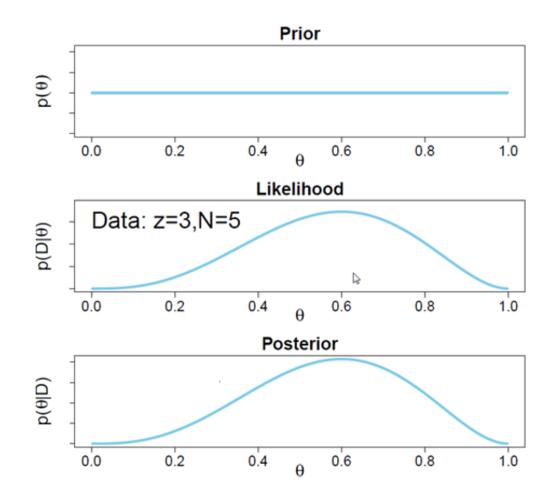
• Recall, for discrete distributions:

$$P(\theta|D) = \frac{P(D|\theta) P(\theta)}{\sum_{i} P(D|\theta_i) P(\theta_i)}$$

• And, for continuous distributions:

$$P(\theta|D) = \frac{P(D|\theta) P(\theta)}{\int P(D|\theta) P(\theta) d\theta}$$

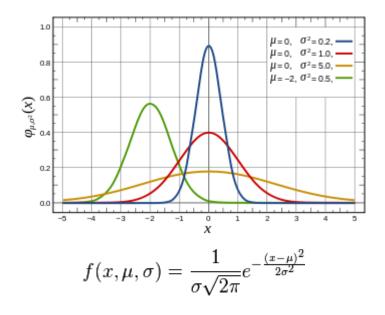
#### **Bayesian** Update



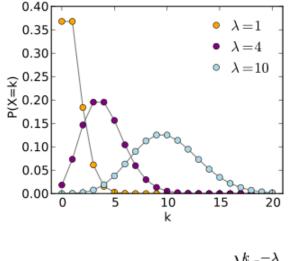
### A Simpler Way

#### Form, Functions and Distributions

Normal (or Gaussian)

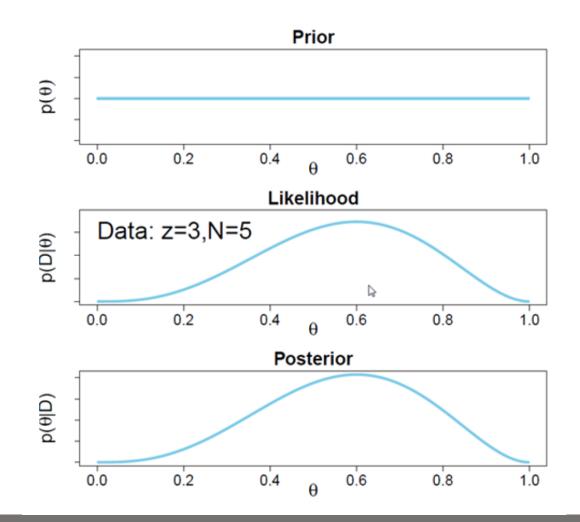


Poisson

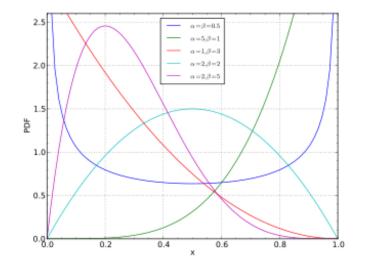


$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^{\kappa} e^{-\lambda}}{k!},$$

#### What form will suit us?



#### Beta Distribution



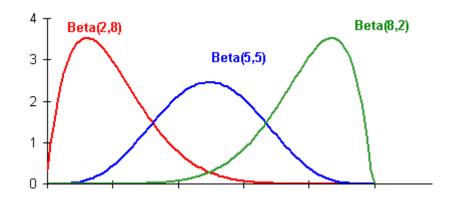
$$f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$



Vadivelu, a famous Tamil comedian. This is one of his great expression - terrified and confused.

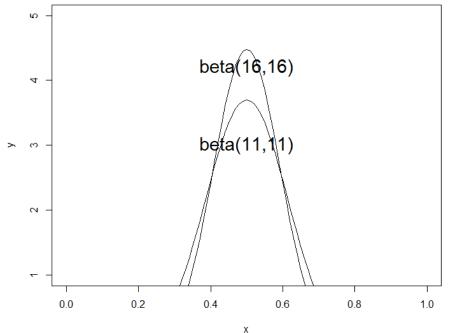
#### Beta Distribution

- Takes two parameters Beta(a,b)
- For now, assume a = #H + 1 and b = #T + 1.
- After 10 flips, we may end up in one of these three:



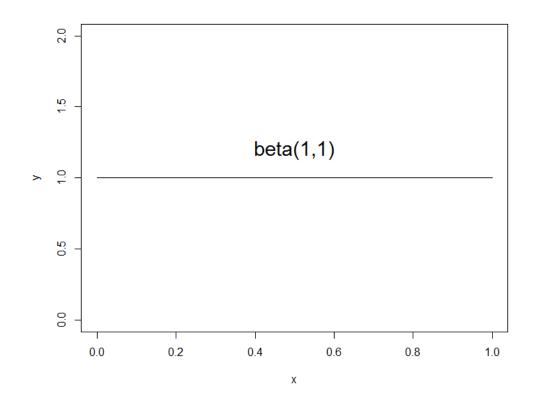
#### Prior and Posterior

#### Let's say we have a Strong Prior - Beta (11,11). What should happen if we see 10 more observations with 5H and 5T?



#### Prior and Posterior...

What if we have not seen any data?



# Conjugate Prior

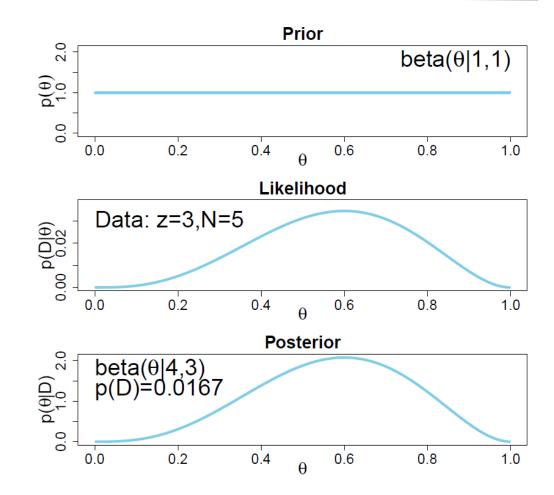
#### So, we see that:



#### Such Priors <u>that have same form</u> are called Conjugate Priors

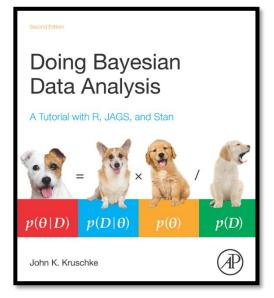
#### Summary

✓Prior ✓Likelihood ✓Posterior ✓ Bayes' Rule ✓ Bernoulli Distribution ✓ Beta Distribution



#### References

#### <u>Book on Doing</u> <u>Bayesian Data</u> <u>Analysis</u> – John K. Kruschke.



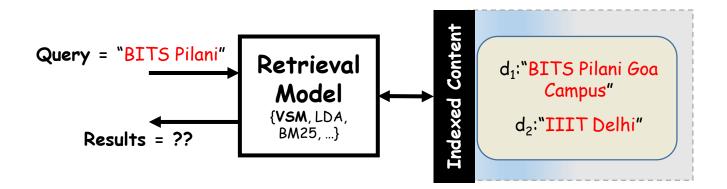
<u>YouTube Video on</u> <u>Statistics 101: The</u> <u>Binomial Distribution</u> – Brandon Foltz.

#### BINOMIAL DISTRIBUTIONS: SALES PERFORMANCE

Brandon C. Foltz, M.Ed. education / training & development / business / tech / math / opinion http://www.botoltz.com/blog Twitter: @BCFoltz YouTube: BCFoltz

# Designing Search Engines

## Which Document to Retrieve?

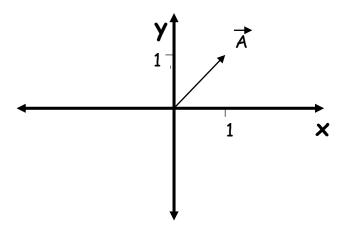


# An Elegant Approach

#### Revisiting Linear Algebra

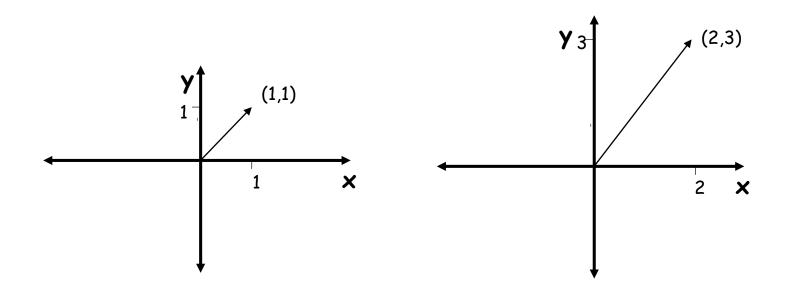
#### Vectors

Geometric entity which has magnitude and direction

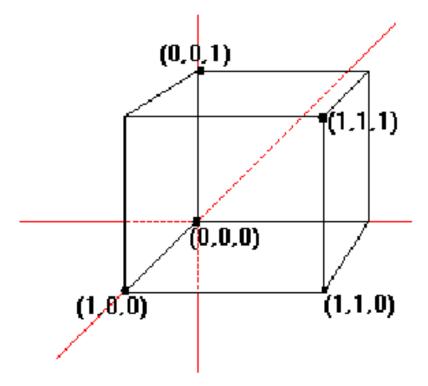


 If (x,y) is our vector of interest, this figure shows A vector = (1,1).

# How is (2,3) Different?



# What is (1,1,1)?

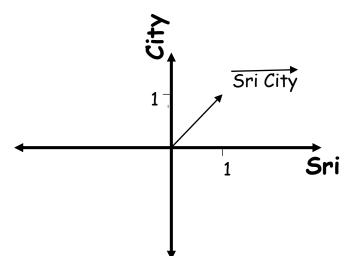


#### Remember!

# A number is just a mathematical object. We give meaning to it!

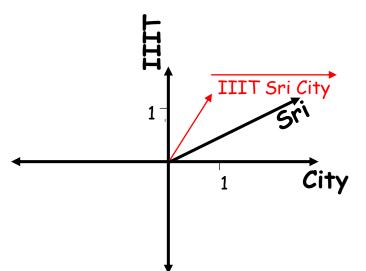
#### Sentences are Vectors

• "Sri City" as a vector



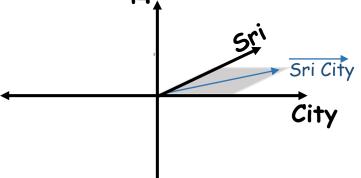
#### Sentences are Vectors

• "IIIT Sri City" is a 3-dimensional vector



#### Sentences are Vectors

• On this 3D space, "Sri City" vector will lie on the x (City) and z (Sri) plane. If (x,y,z) denotes the vector, "Sri City" is (1,0,1).



#### Natural Language Phrases as Vectors

Let query q = "IIIT Sri City". Let document,  $d_1 = "IIIT Sri City"$  and  $d_2 = "IIIT Delhi"$ .

	IIIT	Sri	City	Delhi
q	1	1	1	0
$d_1$	1	1	1	0
d <sub>2</sub>	1	0	0	1

 $q = (1,1,1,0), d_1 = (1,1,1,0) \text{ and } d_2 = (1,0,0,1)$ 

## Quiz

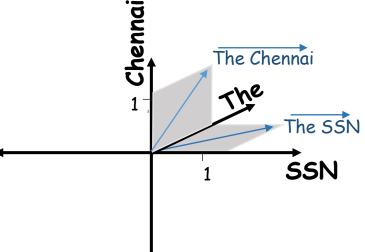
• Considering the following vectors:

	IIIT	Sri	City	Delhi
q	1	1	1	0
$d_1$	1	1	1	0
d <sub>2</sub>	1	0	0	1

- What is the Natural Language (NL) equivalent of (0,1,1,0) ?
- What is the NL equivalent of (1,0,0,1)?
- What is the vector for Delhi?
- What is the NL equivalent of q?

#### Comparing Sentences

• We can compare sentences using the angle between vectors



#### Angle between two vectors

- What is the angle between The and SSN vectors?
- What is the angle between SSN and Chennai vectors?
- What is the angle between The SSN and The SSN vectors?

### Similarity Score

- D1 = "Chennai"
- D2 = "Chennai"
- Quiz
  - On a scale of 0 1, how similar are D1 and D2?
    - 0 → Dissimilar
    - 1 → Identical

#### How to Convert [0 to 90] $\rightarrow$ [1 to 0]

Revisiting Trigonometry

# Converting from "0 - 90" to "1 - 0"

- For convenience, We convert the angles 0 - 90 to values 1 - 0
  - When vectors are perpendicular, we want to output 0.
  - When vectors are same, we want to output 1.





#### 0 - 90 to 1 - 0: How?

	<b>0°</b>	30°	<b>45°</b>	60°	90°
sin $ heta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
<b>cos</b> θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

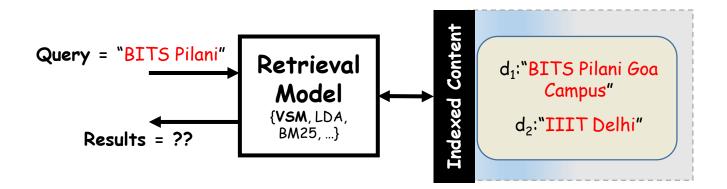
## Back to Trigonometry

• If x and y are non-unit vectors, what is the cosine of angle between them (cos  $\theta$ )?

 $a.b = ||a|| ||b|| \cos(\theta)$ 

similarity(a,b) = 
$$cos(\theta) = \frac{a.b}{||a|| ||b||}$$

#### Which Document to Retrieve?



#### A Boolean Term Document Matrix

	BITS	Pilani	Goa	Campus	IIIT	Delhi
q	1	1	0	0	0	0
$d_1$	1	1	1	1	0	0
d <sub>2</sub>	0	0	0	0	1	1

#### Example

Let query q = "BITS Pilani". Let document,  $d_1 = "BITS Pilani Goa Campus"$  and  $d_2 = "IIIT Delhi"$ .

	BITS	Pilani	Goa	Campus	IIIT	Delhi
q	1	1	0	0	0	0
$d_1$	1	1	1	1	0	0
d <sub>2</sub>	0	0	0	0	1	1

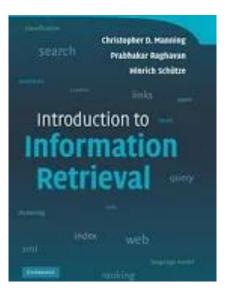
In our VSM, q = (1,1,0,0,0,0),  $d_1 = (1,1,1,1,0,0)$  and  $d_2 = (0,0,0,0,1,1)$ 

similarity(d<sub>1</sub>, q) = 
$$\frac{d_1 \cdot q}{||d_1|| \, ||q||} = \frac{1.1 + 1.1}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{1^2 + 1^2}} = 0.71.$$
  
similarity(d<sub>2</sub>, q) =  $\frac{d_2 \cdot q}{||d_2|| \, ||q||} = 0.$ 

#### References

<u>Introduction to</u> <u>Information Retrieval</u> - Manning, Raghavan and Schutze.

#### <u>Modern Information</u> <u>Retrieval</u> – Yates and Neto.

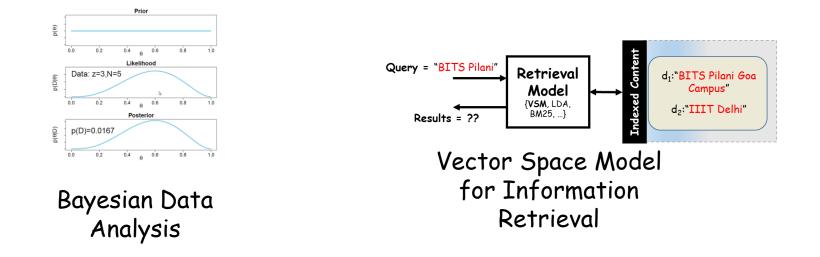




Modern Information Retrieval



#### Summary



Software systems are becoming increasingly complex.

Our ability to design effective abstractions matter!

## Thank You.